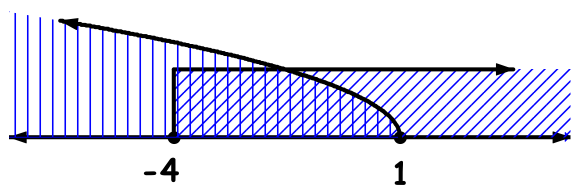
**Math 0350 Practice Test** **Unit Two**
**Chapters 9 and10 part A** **Martin-Gay 4th edition**

Solve the compound inequality. Graph the solution set and write it in interval notation.

1. $3x+1<4$ and $2x+4\geq -4$

**Solution:** The "and" in the problem means that both equations have to be true simultaneously.

$$\begin{array}{lcl} 3x+1<4 & \textit{and} & 2x+4\geq -4 \\ 3x<4-1 & & 2x\geq -4-4 \\ 3x<4 & & 2x\geq -8 \\ x<1 & & x\geq -4 \end{array}$$

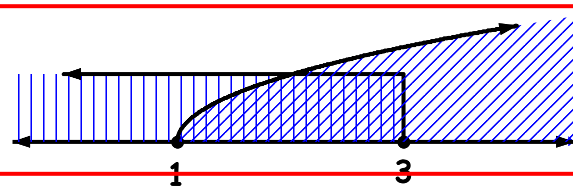



-4 is a valid solution (because of the equal sign), so we use brackets at -4. 1 is not included, so we use parentheses at 1. Both inequalities are true for x in the interval $\boxed{[-4,1)}$.

2. $5x-3>2$ or $-2x\geq -6$

**Solution:** The "or" in the problem means that the solution is where either one of the inequalities is true.

$$\begin{array}{lcl} 5x-3>2 & \textit{or} & -2x\geq -6 \\ 5x>2+3 & & x\leq 3 \\ 5x>5 & & \\ x>1 & & \end{array}$$

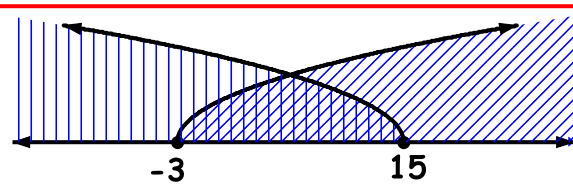



3 is a valid solution (because of the equal sign), so we use brackets at 3. 1 is not included, so we use parentheses at 1. One or both inequalities are true for x in the interval $\boxed{(-\infty,\infty)}$.

3. $-2<\dfrac{x-3}{3}<4$

**Solution:** Solve both inequalities simultaneously. The solution is where both inequalities are true.

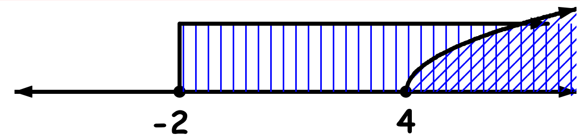
$$-2<\frac{x-3}{3}<4$$
$$-2(3)<\frac{x-3}{3}(3)<4(3)$$
$$-6<x-3<12$$
$$-6+3<x<12+3$$
$$-3<x<15$$




Both inequalities are true for x in the interval $(-3,15)$.

4. $x > 4$ or $x \geq -2$

**Solution:** The "or" in the problem means that the solution is where either one of the inequalities is true. -2 is a valid solution (because of the equal sign), so we use brackets at -2. One or both inequalities are true for x in the interval $\boxed{[-2, \infty)}$




Solve each equation or inequality. Use interval notation when appropriate.

5. $|3x+6|-7=8$

**Solution:** Solve for the absolute value first: $|3x+6|=15$.

Divide the problem into 2 problems: $3x+6=15$ *and* $3x+6=-15$

Solve. There are 2 solutions for most of the absolute value equations:

$$\begin{array}{lcl} 3x+6=15 & & 3x+6=-15 \\ 3x=15-6 & \text{and} & 3x=-15-6 \\ 3x=9 & & 3x=-21 \\ \boxed{x=3} & & \boxed{x=-7} \end{array}$$

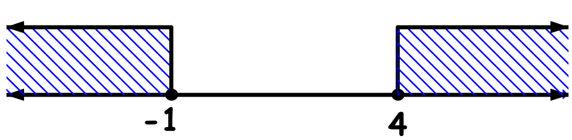
6. $|4x-7|=-5$

**Solution:** Absolute value represents a distance, so this problem doesn't have a solution; the distance cannot be a negative number.

7. $|3-2x| \geq 5$

**Solution:** Absolute value represents a distance, so this problem is $3-2x=?$ so $|3-2x| \geq 5$. This inequality is true when

$$\begin{array}{lcl} 3-2x \geq 5 & & 3-2x \leq -5 \\ -2x \geq 5-3 & \text{and} & -2x \leq -5-3 \\ -2x \geq 2 & & -2x \leq -8 \\ x \leq -1 & & x \leq 4 \end{array}$$




In interval notation: $(-\infty, -1] \cup [4, \infty)$

8. $\left|\frac{2x-1}{3}\right| \le 7$

**Solution:** Absolute value represents a distance, so this problem is $\frac{2x-1}{3} = ?$ so $\left|\frac{2x-1}{3}\right| \le 7$. This inequality is true when $\frac{2x-1}{3} = 7, 6, 5, \ldots, -1, \ldots -7$, because absolute value of those numbers is less or equal to 7. It is not true for -8, because |-8|=8 which is not less or equal to 7. It is also not true for 8, because 8 is not less or equal to 7. The solutions are all the numbers between -7 and 7 included. So we get:

$-7 \le \frac{2x-1}{3} \le 7$ Multiply all the sides with 3:

$-21 \le 2x-1 \le 21$ Add 1 to all the sides:

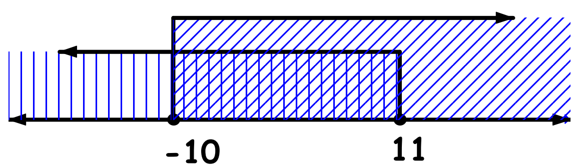
$-21+1 \le 2x-1+1 \le 21+1$

$-20 \le 2x \le 22$ Divide by 2 all the sides:

$\frac{-20}{2} \le \frac{2x}{2} \le \frac{22}{2}$

$\boxed{-10 \le x \le 11}$

So the solution is a number between -10 and 11, included. $\boxed{\text{In interval notation: } [-10, 11]}$




9. $|4-5x| > -6$

**Solution:** Absolute value represents a distance, so this problem is $4-5x = ?$ so $|4-5x| > -6$. This inequality is always true, because the distance is always greater or equal to 0. In interval notation: $\boxed{(-\infty, \infty)}$

10. $|2-3x| = |x+2|$

**Solution:** Divide the problem into 2 problems: $2-3x = +(x+2)$ *and* $2-3x = -(x+2)$

Solve. There are 2 solutions for most of the absolute value equations:

$2-3x = +(x+2)$

$2-3x = x+2$

$-3x = x$

$-3x - x = 0$

$-4x = 0$

$\boxed{x = 0}$

and

$2-3x = -(x+2)$

$2-3x = -x-2$

$-3x = -x-2-2$

$-3x + x = -4$

$-2x = -4$

$\boxed{x = 2}$

11. $|x+3| + 2 < 5$

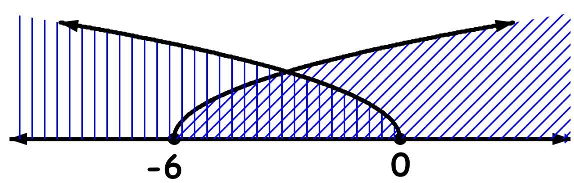
**Solution:** Solve for the absolute value first: $|x+3| < 5-2$; $|x+3| < 3$ .

Absolute value represents a distance, so this problem is $x+3 = ?$ so $|x+3| < 3$. This inequality is true when

$-3 < x+3 < 3$

$-3-3 < x < 3-3$

$-6 < x < 0$



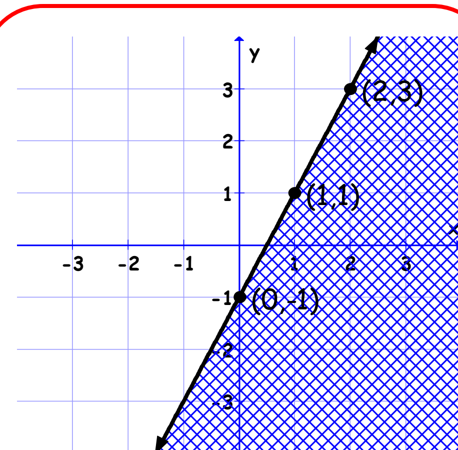

In interval notation: $\boxed{(-6,0)}$

Graph each inequality

12. $y \le 2x - 1$

**Solution:** Step I: Graph the line $y = 2x - 1$ using any of the methods you know.

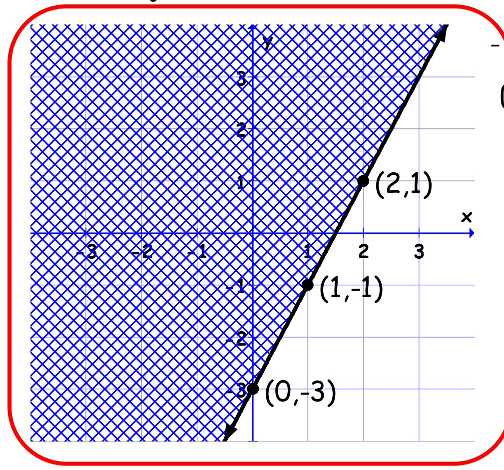
Step II: the solution will be either above the line or below the line. We can check which area is the solution by plugging a random point that does not belong to the line in the inequality: If we select the point (0, 0): $0 \le 2(0) - 1$ *wrong*!




The side that contain the point (0, 0) is not a solution. We shade the other side. Make sure that you graph a bold line, because the line itself is a solution.

13. $4x-2y<6$

**Solution: Step I:** Graph the line $4x-2y=6$ using any of the methods you know.




If we plug values for x = 0, 1, 2, we get the values for y = -3, -1, 1, so points from the line are the points: (0, -3); (1, -1); and (2, 1).

Step II: the solution will be either above the line or below the line.

We can check which area is the solution by plugging a random point that does not belong to the line in the inequality: If we select the point (0, 0):

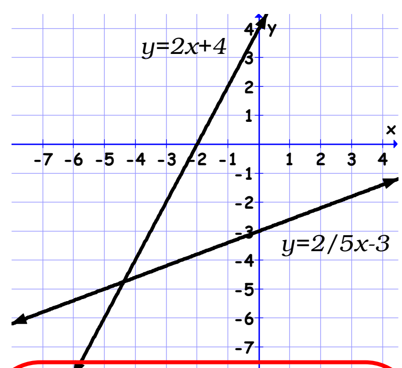
$$4x-2y<6$$
$$4(0)-2(0)<6 \quad correct!$$

The side containing the point (0, 0) is a solution. We shade the side containing the point (0, 0). Make sure you draw a dotted line, because $4x-2y<6$ is a strong inequality which tells us that the points from the line are not solution.

Graph the solution of the following systems of linear inequalities

14. $\begin{cases} 2x-y\ge -4 \\ y>\dfrac{2}{5}x-3 \end{cases}$

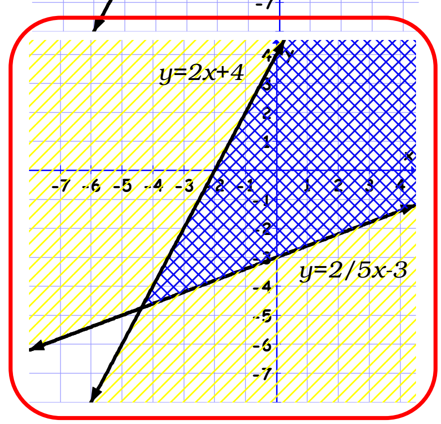
**Solution:** The first inequality has to be graphed with a bold line. The second inequality is a strong inequality, so it has to be graphed with a dotted line:




Perform a point check. Take a point from INSIDE of the areas and plug in in the INEQUALITIES! I will pick the point $(0,0)$

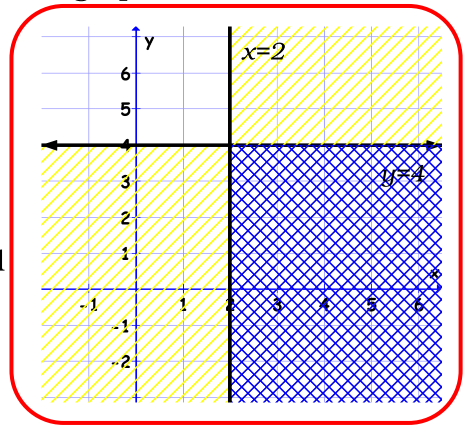
$$\begin{cases} 2(0)-0\ge -4 & \color{red}{correct} \\ 0>-3 & \color{red}{correct} \end{cases} \Rightarrow$$ The point $(0,0)$ belongs to the solution!

The solution is:



15. $\begin{cases} y < 4 \\ x \geq 2 \end{cases}$

**Solution:** The first inequality is a strong inequality, so it has to be graphed with a dotted line; the second inequality has to be graphed with a bold line.




We can either perform a point check or we can shade the areas as described: $y < 4$ are all the points that have y value less than 4, so we shade below the line $y = 4$.

$x \geq 2$ are all the points that have x coordinate greater or equal to 2, so we shade to the right of the line $x = 2$.

The areas overlap in the lower right corner of the graph.

Write each of the following in simple radical form. Assume that all variable expressions represent positive numbers.

16. $\sqrt[3]{64x^9y^6}$

**Solution:** $\sqrt[3]{64x^9y^6} = \sqrt[3]{64}\ \sqrt[3]{x^9}\ \sqrt[3]{y^6} = 4\ x^{\frac{9}{3}}\ y^{\frac{6}{3}} = \boxed{4x^3y^2}$

17. $-\sqrt{49x^3y^{16}}$

**Solution:** $-\sqrt{49x^3y^{16}} = -\sqrt{49}\sqrt{x^3}\sqrt{y^{16}} = -7\ \sqrt{x^2}\sqrt{x}\ y^{\frac{16}{2}} = \boxed{-7\,x\,y^8\sqrt{x}}$

18. $\sqrt[4]{16x^{10}y^5}$

**Solution:** $\sqrt[4]{16x^{10}y^5} = \sqrt[4]{16}\ \sqrt[4]{x^{10}}\ \sqrt[4]{y^5} = 2\ \sqrt[4]{x^8}\sqrt[4]{x^2}\sqrt[4]{y^4}\sqrt[4]{y} = 2\ x^{\frac{8}{4}}\ y^{\frac{4}{4}}\ \sqrt[4]{x^2}\sqrt[4]{y} = \boxed{2\ x^2y\ \sqrt[4]{x^2y}}$

19. $\sqrt[5]{(x-2)^5}$

**Solution:** $\sqrt[5]{(x-2)^5} = (x-2)^{\frac{5}{5}} = (x-2)^1 = \boxed{x-2}$

20. $\sqrt[3]{\dfrac{54x^{13}y^5}{2x^4y^2}}$

**Solution:** $\sqrt[3]{\dfrac{54x^{13}y^5}{2x^4y^2}} = \sqrt[3]{27x^{13-4}y^{5-2}} = \sqrt[3]{27x^9y^3} = \sqrt[3]{27}\sqrt[3]{x^9}\sqrt[3]{y^3} = 3x^{\frac{9}{3}}y^{\frac{3}{3}} = \boxed{3x^3y}$

Simplify each expression. Assume that all variables represent positive numbers. Exponents in the final answer should be positive.

21. $\left(\dfrac{x^{\frac{2}{3}}}{y^{\frac{-1}{3}}}\right)^6$

**Solution:** $\left(\dfrac{x^{\frac{2}{3}}}{y^{\frac{-1}{3}}}\right)^6 = \dfrac{\left(x^{\frac{2}{3}}\right)^6}{\left(y^{\frac{-1}{3}}\right)^6} = \dfrac{x^{\frac{2}{3}\cdot 6}}{y^{\frac{-1}{3}\cdot 6}} = \dfrac{x^4}{y^{-2}} = \boxed{x^4y^2}$

22. $\left(\dfrac{x^{\frac{1}{2}}}{y}\right)^{-2}$

**Solution:** $\left(\dfrac{x^{\frac{1}{2}}}{y}\right)^{-2} = \dfrac{\left(x^{\frac{1}{2}}\right)^{-2}}{\left(y^1\right)^{-2}} = \dfrac{x^{\frac{1}{2}(-2)}}{y^{1\cdot(-2)}} = \dfrac{x^{-1}}{y^{-2}} = \boxed{\dfrac{y^2}{x}}$

23. $a^{\frac{2}{3}}(a^{\frac{1}{3}} - 2a^{\frac{4}{3}})$

**Solution:** $a^{\frac{2}{3}}(a^{\frac{1}{3}} - 2a^{\frac{4}{3}}) = a^{\frac{2}{3}}(a^{\frac{1}{3}}) + a^{\frac{2}{3}}(-2a^{\frac{4}{3}}) = a^{\frac{2}{3}+\frac{1}{3}} - 2a^{\frac{2}{3}+\frac{4}{3}} = a^{\frac{3}{3}} - 2a^{\frac{6}{3}} = \boxed{a - 2a^2}$

Perform the indicated operations. Assume that all variables represent positive numbers.

24. $\sqrt[3]{25x^2y^4} \cdot \sqrt[3]{5x^7y^8}$

**Solution:** $\sqrt[3]{25x^2y^4} \cdot \sqrt[3]{5x^7y^8} = \sqrt[3]{25x^2y^4 5x^7y^8} = \sqrt[3]{125x^{2+7}y^{4+8}} = \sqrt[3]{5^3x^9y^{12}} = 5\,x^{\frac{9}{3}}\,y^{\frac{12}{3}} = \boxed{5\,x^3\,y^4}$

25. $\dfrac{6\sqrt{a^5b}}{\sqrt{4a^2b^3}}$

**Solution:** $\dfrac{6\sqrt{a^5b}}{\sqrt{4a^2b^3}} = 6\sqrt{\dfrac{a^5b}{4a^2b^3}} = 6\sqrt{\dfrac{a^{5-2}}{4b^{3-1}}} = 6\sqrt{\dfrac{a^3}{4b^2}} = \dfrac{6\sqrt{a^3}}{\sqrt{4b^2}} = \dfrac{6\sqrt{a^2}\sqrt{a}}{\sqrt{4}\sqrt{b^2}} = \dfrac{6a\sqrt{a}}{2b} = \boxed{\dfrac{3a\sqrt{a}}{b}}$

26. $3\sqrt{32x^2} + 5x\sqrt{8}$

**Solution:** $3\sqrt{32x^2} + 5x\sqrt{8} = 3\sqrt{16(2)x^2} + 5x\sqrt{4(2)} = 3\sqrt{16}\sqrt{2}\sqrt{x^2} + 5x\sqrt{4}\sqrt{2} =$

$= 3\cdot 4x\sqrt{2} + 5x\cdot 2\sqrt{2} = 12x\sqrt{2} + 10x\sqrt{2} = \boxed{22x\sqrt{2}}$

27. $(2-3\sqrt{3})(2+3\sqrt{3})$

**Solution:**

$$(2-3\sqrt{3})(2+3\sqrt{3})=2\cdot 2+2\cdot 3\sqrt{3}-3\sqrt{3}\cdot 2-3\sqrt{3}\cdot 3\sqrt{3}=4+6\sqrt{3}-6\sqrt{3}-9\left(\sqrt{3}\right)^2=4-9\cdot 3=4-27\boxed{=-23}$$

28. $(\sqrt{7}+\sqrt{2})^2$

**Solution:**

$$(\sqrt{7}+\sqrt{2})^2=(\sqrt{7}+\sqrt{2})(\sqrt{7}+\sqrt{2})=\sqrt{7}\sqrt{7}+\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}+\sqrt{2}\sqrt{2}=7+\sqrt{14}+\sqrt{14}+2=\boxed{9+2\sqrt{14}}$$

29. $\sqrt[3]{54x^4}+4x\ \sqrt[3]{16x}$

**Solution:** $\sqrt[3]{54x^4}+4x\ \sqrt[3]{16x}=\sqrt[3]{27\cdot 2\cdot x^3\cdot x}+4x\ \sqrt[3]{8\cdot 2\cdot x}=\sqrt[3]{27}\,\sqrt[3]{x^3}\,\sqrt[3]{2\cdot x}+4x\ \sqrt[3]{8}\,\sqrt[3]{2\cdot x}=$

$$=3x\sqrt[3]{2x}+4x\cdot 2\sqrt[3]{2x}=3x\sqrt[3]{2x}+8x\sqrt[3]{2x}=\boxed{11x\sqrt[3]{2x}}$$

30. $\sqrt{45}-\sqrt{20}$

**Solution:** $\sqrt{45}-\sqrt{20}=\sqrt{5\cdot 9}-\sqrt{5\cdot 4}=\sqrt{5}\sqrt{9}-\sqrt{5}\sqrt{4}=3\sqrt{5}-2\sqrt{5}=\boxed{\sqrt{5}}$

31. $\left(4\sqrt{3}-3\sqrt{2}\right)\left(5\sqrt{3}+3\sqrt{2}\right)$

**Solution:** $\left(4\sqrt{3}-3\sqrt{2}\right)\left(5\sqrt{3}+3\sqrt{2}\right)=4\sqrt{3}\cdot 5\sqrt{3}+4\sqrt{3}\cdot 3\sqrt{2}-3\sqrt{2}\cdot 5\sqrt{3}-3\sqrt{2}\cdot 3\sqrt{2}=$

$$=20\left(\sqrt{3}\right)^2+12\sqrt{6}-15\sqrt{6}-9\left(\sqrt{2}\right)^2=20\cdot 3-3\sqrt{6}-9\cdot 2=60-18-3\sqrt{6}=\boxed{42-3\sqrt{6}}$$

32. Find the distance between the points (-5,8) and (1, 16).

**Solution:** The distance formula is: $d=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}$

$x_1=-5;\quad x_2=1;\quad y_1=8;\quad y_2=16$

$d=\sqrt{\left(-5-1\right)^2+\left(8-16\right)^2}$

Don't forget to use parentheses properly!!!

$d=\sqrt{\left(-6\right)^2+\left(-8\right)^2}$

$d=\sqrt{36+64}=\sqrt{100}=10$

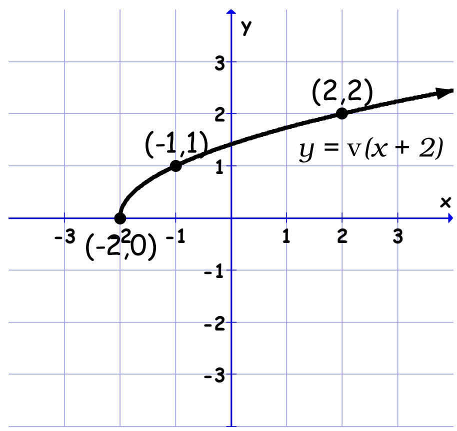
The answer is 10 units.

Sketch a graph of each of the following functions. List the domain and range of the function. Label 3 exact points.

33. $f(x)=\sqrt{x+2}$

**Solution:** We always pick the first point to be at the corner of the graph (this is the x-value that makes the expression under the square root = 0. In this case this is x+2 = 0, or x =- 2. Then we pick two points from each side of the corner point and calculate the y values at them. Notice that the square root graph goes one way only so we will only receive y-values to one of the sides of this function. I picked x = 2, because it is easy to take root of 4. If I pick x = 0, or 1, then I'll have to take a root of 2 or 3, which are not exact numbers:

| $x$ | $f(x)=\sqrt{x+2}$ |
|---|---|
| $-4$ | $\sqrt{-4+2}=\sqrt{-2}$ |
| $-3$ | $\sqrt{-3+2}=\sqrt{-1}$ |
| $-2$ | $\sqrt{-2+2}=\sqrt{0}=0$ |
| $-1$ | $\sqrt{-1+2}=\sqrt{1}=1$ |
| $2$ | $\sqrt{2+2}=\sqrt{4}=2$ |




The corner point is at (-2, 0).

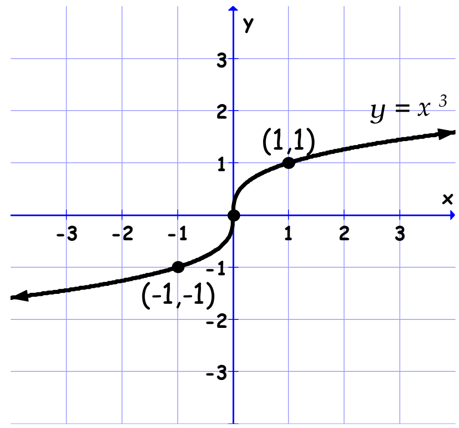
Domain: $[-2,\infty)$ (because we cannot plug values for x less than -2; if we do we get negative numbers under the root)

Range: $[0,\infty)$ (because there is no y-value lower than 0 on the graph; it is a closed interval, because there is a point that contains y=0.)

34. $g(x)=\sqrt[3]{x}$

**Solution:** We always pick the first point to be the x-value that makes the expression under the root = 0. In this case this is x = 0. Then we pick two points from each side of the vertex value and calculate the y values at them. If we work correctly the y values are going to be opposite numbers for each pair of x or the points are going to be symmetrical in respect of the origin.

| $x$ | $g(x)=\sqrt[3]{x}$ |
|---|---|
| $-8$ | $\sqrt[3]{-8}=-2$ |
| $-1$ | $\sqrt[3]{-1}=-1$ |
| $0$ | $\sqrt[3]{0}=0$ |
| $1$ | $\sqrt[3]{1}=1$ |
| $8$ | $\sqrt[3]{8}=2$ |




Domain: $(-\infty,\infty)$ ; Range: $(-\infty,\infty)$